

The Working Backwards heuristic

The Working Backwards heuristic is a problem-solving strategy usually used to solve Mathematics questions that involve a series of related events.

To apply this heuristic, one needs to use the number given at the end state of the problem situation and follow the sequence of events from the last event to the first event to arrive at the initial state.

How do we identify questions that need working backwards?

- 1) In the question, the number of people or objects at first is unknown.
- 2) A series of events takes place.
- 3) The final number is given at the end of the question situation.
- 4) The question requires to find the beginning number.

Let's start with a simple question situation:

1.
 There were some passengers on a bus.
 At bus stop A, 5 passengers alighted and 8 passengers boarded.
 At bus stop B, 6 passengers alighted.
 Then there were 70 passengers.
 How many passengers were there on the bus at first?

Using the 4-step POLYA processes to solve the problem:

Step 1 - Understand the problem situation and organise the information in a table:

passengers at	alight	board	number
bus stop A	5	8	
bus stop B	6		
the bus in the end			70

Step 2 - Action plan - Work backwards

Use the end number and work backwards to arrive at the number of passengers at first.

Step 3 - Solve the problem

Start with 70 passengers

passengers who boarded/ alighted	calculate backwards
6 alighted	$70 + 6 = 76$
8 boarded	$76 - 8 = 68$
5 alighted	$68 + 5 = 73$

Answer: There were 73 passengers on the bus at first.

Step 4 - Check the solution

Start from the beginning:

passengers at	alight	board
bus stop A	$73 - 5 = 68$	$68 + 8 = 76$
bus stop B	$76 - 6 = 70$	

✓ There were 70 passengers then.

Your turn to try with this similar type question scenario:

2.

A train was two stops away from Sunli interchange.

At train stop X, 21 passengers boarded and 12 passengers alighted.

At train stop Y, 35 passengers alighted.

At Sunli interchange, all 90 passengers alighted.

How many passengers were there in the train before it arrived at train stop X?

Check your working steps with this solution:

Using the 4-step POLYA processes to solve the problem:

Step 1 - Understand the problem situation and organise the information in a table:

passengers at	board	alight
train stop X	21	12
train stop Y		35
Sunli interchange		90

Step 2 –Action plan - Work backwards

Use the end number and work backwards to arrive at the number of passengers at first.

Step 3 – Solve the problem

Start with 90 passengers

passengers who boarded/ alighted	calculate backwards
35 alighted	$90 + 35 = 125$
12 alighted	$125 + 12 = 137$
21 boarded	$137 - 21 = 116$

Answer: There were 116 passengers in the train before it arrived at train stop X.

Step 4 – Check the solution

Start from the beginning:

passengers at	alight	board
train stop X	$116 + 21$ $= 137$	$137 - 12$ $= 125$
train stop Y	$125 - 35$ $= 90$	

✓ 90 passengers alighted at Sunli interchange.

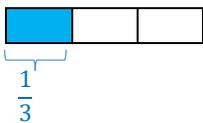
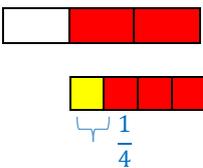
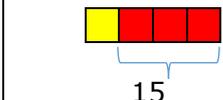
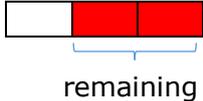
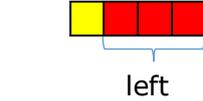
Next we involve questions with fractions

3.

Mrs Ow bought $\frac{1}{3}$ of the mangoes in a fruit crate. Then Mr Toh bought $\frac{1}{4}$ of the remaining mangoes. 15 mangoes were left and Mr Sim bought them. How many mangoes were there in the crate at first?

Using the 4-step POLYA processes to solve the problem:

Step 1 - Understand the problem situation and organise the information in a diagram table:

	Mr Ow	Mr Toh	Mr Sim
bought			
remaining			

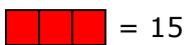
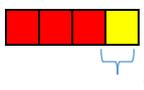
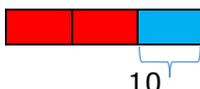
Step 2 - Action plan - Work backwards

Use the end number and work backwards to find the number of mangoes in the crate at first.

Step 3 - Solve the problem

Start with 15 mangoes

3 units = 15

	bought	Calculate backwards
Mr Sim	 = 15	3 units = 15 1 unit = $15 \div 3 = 5$
Mr Toh		4 units = $4 \times 5 = 20$
Mr Ow		2 units = 20 1 unit = $20 \div 2 = 10$ 3 units = $3 \times 10 = 30$

Answer: There were 30 mangoes in the crate at first.

Step 4 - Check the solution

Start from the beginning:

	Mr Ow	Mr Toh	Mr Sim	total
mangoes bought	10	5	15	30

✓ There were 30 mangoes in the crate at first.

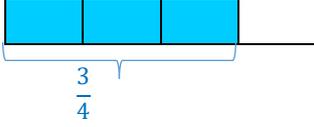
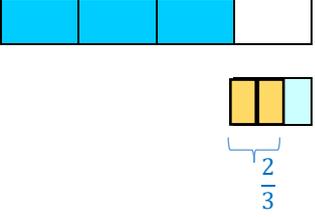
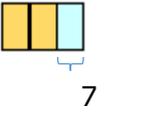
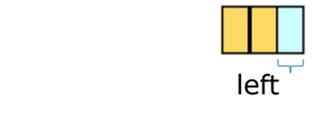
Your turn to try with a similar type question scenario:

4. $\frac{3}{4}$ of the potatoes in a shop were sold on Monday. $\frac{2}{3}$ of the remaining potatoes were sold on Tuesday. Then 7 potatoes were left and they were sold on Wednesday. How many potatoes were there in the shop at first?

Check your working steps with this solution:

Using the 4-step POLYA processes to solve the problem:

Step 1 - Understand the problem situation and organise the information in a diagram table:

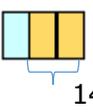
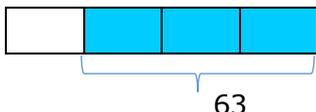
	Monday	Tuesday	Wednesday
sold			
remaining			

Step 2 -Action plan - Work backwards

Use the end number and work backwards to find the number of potatoes in the shop at first.

Step 3 - Solve the problem

Start with 7 potatoes

	bought	Calculate backwards
Wednesday	 = 7	1 unit = 7
Tuesday	 14	2 units = $2 \times 7 = 14$ 3 units = $3 \times 7 = 21$
Monday	 63	1 unit = 21 3 units = $3 \times 21 = 63$ 4 units = $4 \times 21 = 84$

Answer: There were 84 potatoes in the shop at first.

Step 4 - Check the solution

Start from the beginning:

	Monday	Tuesday	Wednesday	total
potatoes bought	63	14	7	84

✓ There were 84 potatoes in the shop at first.

Finally we involve questions with percentage

5.

An electronics store was having a sale.

Buy 2 items and get the 2nd item at 20% less!

The usual price of the 2nd item must be equal to or lower than the usual price of the 1st item.

Mrs Lim bought a water heater and a table fan during the sale.

The water heater cost \$9.60 more.

She paid \$38.40 for both items. What was the price of the table fan before the discount?

Using the 4-step POLYA processes to solve the problem:

Step 1 - Understand the problem situation and organise the information in a diagram table:

bought		total spent
1 st item - water heater		\$38.40
2 nd item - table fan @ 20% less		

Step 2 -Action plan - Work backwards

Use the end total amount and work backwards to find the price of the table fan before 20% discount.

Step 3 - Solve the problem

Start with \$38.40

$$\$38.40 - \$9.60 = \$28.80$$

$$2 \text{ units} = \$28.80$$

1 unit = $\$28.80 \div 2 = \14.40 The price of the table fan was \$14.40 after 20% discount.

$$100\% - 20\% = 80\%$$

The table fan was sold at 80% of its usual price.

$$80\% = \$14.40$$

$$10\% = \$14.40 \div 8 = \$1.80$$

$$100\% \text{ (usual price of the table fan)} = 10 \times \$1.80 = \$18$$

Answer: The usual price of the table fan was \$18.

Usual price		total before discount
1 st item - water heater		\$42
2 nd item - table fan		

Step 4 – Check the solution

Start from the beginning:

	water heater	table fan	total
100%	\$24	\$18	\$42
less 20%	X	20% x \$18 = \$3.60	\$42 - \$3.60 = \$38.40

✓ The usual price of the table fan was \$18.

Your turn to try with this similar type question scenario:

6. A toy store was having a sale.

Buy 1st item at 20% off!
Buy 2nd item at 50% off!

The usual price of the 2nd item must be equal to or lower than the usual price of the 1st item.

Mydin bought a train set and a hula hoop during the sale. The train set cost \$11.40 more. She paid \$29.40 for both items. What was the price of the train set before the discount?

Check your working steps with this solution:

Using the 4-step POLYA processes to solve the problem:

Step 1 - Understand the problem situation and organise the information in a diagram table:

bought		total spent
1 st item – train set @ 20% discount		\$29.40
2 nd item – hula hoop @ 50% discount		

Step 2 –Action plan - Work backwards

Use the end total amount and work backwards to find the price of the hula hoop before 50% discount and the price of the train set before 20% discount.

Step 3 – Solve the problem

Start with \$29.40

$$\$29.40 - \$11.40 = \$18$$

$$2 \text{ units} = \$18$$

$$1 \text{ unit} = \$18 \div 2 = \$9 \quad \text{The price of the hula hoop was } \$9 \text{ after } 50\% \text{ discount.}$$

$$\$9 + \$11.40 = \$20.40$$

The price of the train set was \$20.40 after 20% discount.

$$100\% - 20\% = 80\%$$

The train set was sold at 80% of its usual price.

$$80\% = \$20.40$$

$$10\% = \$20.40 \div 8 = \$2.55$$

$$100\% \text{ (usual price of the train set)} = 10 \times \$2.55 = \$25.50$$

Answer: The usual price of the train set was \$25.50.

Usual price		total before discount
1 st item – train set		\$43.50
2 nd item – hula hoop		

Step 4 – Check the solution

Start from the beginning:

	Train set	Hula hoop	total
100%	\$25.50	\$18	\$43.50
	Less 20% = 20% x \$25.50 = \$5.10	Less 50% = 50% x \$18 = \$9	\$43.50 - \$14.10 = \$29.40
Price after discount	\$20.40	\$9	\$29.40

✓ The usual price of the train set was \$25.50

In summary, to apply the Working Backwards strategy:

'start the solution process with the number given at the end state of the problem situation and follow the sequence of events from the last event to the first event to arrive at the initial state.'

Through constant usage of the Working Backwards strategy, solving problems can become simple, precise and fast.

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